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Two-dimensional electron gas in a uniform magnetic field in the presence of a δ-impurity

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Abstract. The density of states and the Hall conductivity of a two-dimensional electron gas in a uniform magnetic field and in the presence of a δ-impurity are exactly calculated using elementary field theoretic techniques. The impurity creates one localized state per Landau level, but the Hall conductivity is unaffected. Our treatment is explicitly gauge invariant, and can be easily adapted to other problems involving zero-range potentials.

1. Introduction

One of the most puzzling features of the quantum Hall effect is the apparent insensitivity of the quantization of the Hall conductivity σ_H (in multiples of e^2/h) with respect to type of host material, geometry of sample, presence of impurities, etc. Prange [1, 2] was probably the first to address the question of the influence of impurities on the quantization of σ_H. For a two-dimensional electron gas in crossed electric and magnetic fields in the presence of a δ-impurity, he showed that a localized state exists, which carries no current, while the remaining nonlocalized states carry an extra Hall current which exactly compensates for the part not carried by the localized state.

Notwithstanding Prange's claim that his calculation was exact, he had in fact to resort to some approximations. In part, this occurred because he worked with a finite (although small) electric field, and in part because he used a δ-function potential, which is too 'strong' in two (or more) dimensions [3], even in the presence of a magnetic field. As shown explicitly by Perez and Coutinho [4], if one solves the Schrödinger equation for a square well of radius a and depth $V_0(a) \sim a^{-2}$, one finds a bound state whose energy $E_b \to -\infty$ when $a \to 0$. In order for E_b to remain finite in the limit $a \to 0$, the depth of the well must diverge more slowly than a^{-2}; explicitly, $V_0(a) \sim 1/a^2 \ln(a/R)$ (R is some constant with dimension of length). Working with this regularized version of the δ-function, they managed to find a spectrum similar to the one found by Prange.

The purpose of this paper is to revisit this problem using elementary field theoretic techniques. There are a couple of reasons for doing things this way: (i) it is very easy to find the Feynman propagator in the presence of a δ-impurity [5, 6], and (ii) the density of states and the conductivity tensor can be computed exactly, in an explicitly gauge-invariant

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way. The very singular nature of the δ-function potential in two dimensions shows up in our treatment as an infinity in the propagator, but to deal with it is a very simple exercise in renormalization [6].

Our calculations essentially confirm Prange's results.

2. The Feynman propagator

Let us consider an electron gas in two dimensions in a uniform magnetic field, in the presence of a δ-function potential at the origin. Its Lagrangian density is given by (we use units such that $m = \hbar = c = 1$)

$$\mathcal{L} = \psi^\dagger(\mathrm{i}\partial_t - H + \mu)\psi. \tag{1}$$

The 'one-particle Hamiltonian' H can be split into two parts: $H = H_0 + V(\boldsymbol{x})$, where

$$H_0 = -\tfrac{1}{2}(\boldsymbol{\nabla} - \mathrm{i}e\boldsymbol{A})^2 \qquad V(\boldsymbol{x}) = \lambda\delta^2(\boldsymbol{x}). \tag{2}$$

The vector potential $\boldsymbol{A}$ generates a uniform magnetic field B $(= \partial_1 A_2 - \partial_2 A_1)$, and μ denotes the chemical potential.

As we shall see in the next section, the particle density and the conductivity of the system can be computed once one knows the Feynman propagator, which satisfies the following equation ($x \equiv (t, \boldsymbol{x})$):

$$(\mathrm{i}\partial_t - H + \mu)_x G(x, x') = \delta^3(x - x'). \tag{3}$$

Since H is time independent, we can look for a solution of (3) in the form

$$G(x, x') = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \mathrm{e}^{-\mathrm{i}\omega(t-t')} G(\omega; \boldsymbol{x}, \boldsymbol{x}'). \tag{4}$$

This in fact is a solution of (3), provided $G(\omega; \boldsymbol{x}, \boldsymbol{x}')$ is a solution of

$$(\omega - H + \mu)_x G(\omega; \boldsymbol{x}, \boldsymbol{x}') = \delta^2(\boldsymbol{x} - \boldsymbol{x}'). \tag{5}$$

This can be solved in the usual way as

$$G(\omega; \boldsymbol{x}, \boldsymbol{x}') = \sum_n \frac{\psi_n(\boldsymbol{x})\psi_n^*(\boldsymbol{x}')}{\omega - E_n + \mu} \tag{6}$$

where E_n and $\psi_n(\boldsymbol{x})$ are the eigenvalues and eigenfunctions of H, respectively. Since H is a Hermitian operator, its eigenvalues are real, so that a prescription must be provided to deal with the poles of $G(\omega; \boldsymbol{x}, \boldsymbol{x}')$ when one performs the integral over ω in (4). For the Feynman propagator, this amounts [7] to a deformation of the integration contour in the complex ω-plane as indicated in figure 1.

Defining the 'unperturbed' propagator $G_0(\omega; \boldsymbol{x}, \boldsymbol{x}')$ as the solution of (5) with $V = 0$, we can formally solve for G as

$$G = G_0 + G_0 V G_0 + G_0 V G_0 V G_0 + \cdots. \tag{7}$$

(The product of two operators F and G is defined as $(FG)(\boldsymbol{x}, \boldsymbol{x}') \equiv \int \mathrm{d}^2 y\, F(\boldsymbol{x}, \boldsymbol{y})G(\boldsymbol{y}, \boldsymbol{x}')$, and $V(\boldsymbol{x}, \boldsymbol{x}')$ stands for $V(\boldsymbol{x})\delta^2(\boldsymbol{x} - \boldsymbol{x}')$.) Because of the very simple form of V, all the integrals in (7) can be performed exactly, and the series can be summed in closed form (we drop the dependence on ω for simplicity):

$$\begin{aligned} G(\boldsymbol{x}, \boldsymbol{x}') &= G_0(\boldsymbol{x}, \boldsymbol{x}') + \lambda G_0(\boldsymbol{x}, \boldsymbol{0}) \sum_{n=0}^{\infty} [\lambda G_0(\boldsymbol{0}, \boldsymbol{0})]^n G_0(\boldsymbol{0}, \boldsymbol{x}') \\ &= G_0(\boldsymbol{x}, \boldsymbol{x}') + \frac{G_0(\boldsymbol{x}, \boldsymbol{0})G_0(\boldsymbol{0}, \boldsymbol{x}')}{\frac{1}{\lambda} - G_0(\boldsymbol{0}, \boldsymbol{0})}. \end{aligned} \tag{8}$$

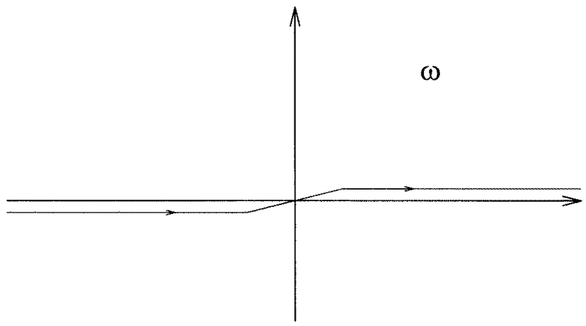

Figure 1. Integration contour in the complex ω-plane used in the definition of the Feynman propagator.

One can verify, by direct substitution in equation (5) (with $V = 0$), that the 'unperturbed' Feynman propagator is given by (eB is assumed positive)

$$G_0(\omega; \boldsymbol{x}, \boldsymbol{x}') = \frac{eB}{2\pi} M(\boldsymbol{x}, \boldsymbol{x}') \mathrm{e}^{-eB(\boldsymbol{x}-\boldsymbol{x}')^2/4} \sum_{n=0}^{\infty} \frac{L_n\left(eB(\boldsymbol{x}-\boldsymbol{x}')^2/2\right)}{\omega - (n+\frac{1}{2})eB + \mu} \tag{9}$$

where $L_n(z)$ is a Laguerre polynomial [8] and $M(\boldsymbol{x}, \boldsymbol{x}')$ is a gauge-dependent factor, which can be written in a gauge covariant way as

$$M(\boldsymbol{x}, \boldsymbol{x}') = \exp\left\{\mathrm{i}e \int_{\boldsymbol{x}'}^{\boldsymbol{x}} \boldsymbol{A}(\boldsymbol{z}) \cdot \mathrm{d}\boldsymbol{z}\right\} \tag{10}$$

the integral being performed along a straight line connecting $\boldsymbol{x}'$ to $\boldsymbol{x}$.

Given the explicit form of $G_0(\omega; \boldsymbol{x}, \boldsymbol{x}')$, (8) gives the solution of equation (5) but, as it stands, it is meaningless: the denominator of the second term on the r.h.s. is logarithmically divergent. However, this divergence can be absorbed in a redefinition of the 'coupling constant' λ: introducing a convergence factor $\mathrm{e}^{-\alpha n}$ in the sum over Landau levels, one finds ($z \equiv \frac{1}{2} - (\omega + \mu)/eB$)

$$\begin{aligned} G_0(\omega; \mathbf{0}, \mathbf{0}) &= \frac{eB}{2\pi} \sum_{n=0}^{\infty} \frac{\mathrm{e}^{-\alpha n}}{\omega - (n+\frac{1}{2})eB + \mu} \\ &= -\frac{1}{2\pi}\left[\sum_{n=0}^{\infty} \mathrm{e}^{-\alpha n}\left(\frac{1}{z+n} - \frac{1}{n+1}\right) + \sum_{n=0}^{\infty} \frac{\mathrm{e}^{-\alpha n}}{n+1}\right] \\ &\approx \frac{1}{2\pi}[\gamma + \psi(z) + \ln\alpha] \qquad (\alpha \to 0^+) \end{aligned} \tag{11}$$

$\psi(z)$ denotes the digamma function and $\gamma = 0.577\ldots$ is Euler's constant [9]. Now, we define a renormalized 'coupling constant' λ_R as

$$\frac{1}{\lambda_R} = \frac{1}{\lambda} - \frac{1}{2\pi}(\gamma + \ln\alpha) \tag{12}$$

and make λ depend on α in such a way that λ_R remains finite† in the limit $\alpha \to 0^+$.

† Note that, for this to be possible, λ must be negative, i.e. the potential is attractive. If one starts with a repulsive δ-potential, there is no way to deal with the divergent terms in the perturbative expansion of G. This reflects the fact that in dimensions $D \geqslant 2$ a repulsive δ-potential expels the S-waves from the Hilbert space [3]. In this paper we consider only the attractive case.

In terms of the renormalized 'coupling constant,' equation (8) becomes

$$G(\boldsymbol{x},\boldsymbol{x}') = G_0(\boldsymbol{x},\boldsymbol{x}') + \frac{G_0(\boldsymbol{x},\boldsymbol{0})G_0(\boldsymbol{0},\boldsymbol{x}')}{\frac{1}{\lambda_R} - \frac{1}{2\pi}\psi\left(\frac{1}{2} - \frac{\omega+\mu}{eB}\right)} \equiv G_0(\boldsymbol{x},\boldsymbol{x}') + G_1(\boldsymbol{x},\boldsymbol{x}') \tag{13}$$

which is now well defined.

3. Particle density, localized states and Hall conductivity

3.1. Particle density

The particle density is given by [7]

$$n(x) = -\mathrm{i} \lim_{t'\to t+0} G(t,\boldsymbol{x};t',\boldsymbol{x}). \tag{14}$$

The 'unperturbed' part of $n(x)$ is position independent:

$$n_0 = -\mathrm{i} \lim_{\alpha\to 0^+} \lim_{\varepsilon\to 0^+} \int \frac{\mathrm{d}\omega}{2\pi} \mathrm{e}^{\mathrm{i}\omega\varepsilon} \frac{eB}{2\pi} \sum_{n=0}^{\infty} \frac{\mathrm{e}^{-\alpha n}}{\omega - (n+\frac{1}{2})eB + \mu}. \tag{15}$$

Because of the exponential in front of the sum, one can close the contour depicted in figure 1 with a semicircle of infinite radius in the upper half-plane, and use residues to evaluate the integral. The result, after taking the limits $\varepsilon \to 0^+$ and $\alpha \to 0^+$ (in this order), is

$$n_0 = \frac{eB}{2\pi} \sum_{n=0}^{\infty} \theta(\mu - (n+\tfrac{1}{2})eB) \tag{16}$$

where $\theta(x)$ is the Heaviside step function.

With respect to the 'perturbed' part of $n(x)$, it is easier to compute $N_1 \equiv \int \mathrm{d}^2x\, n_1(x)$, where $n_1(x) \equiv n(x) - n_0$:

$$N_1 = -\mathrm{i} \lim_{\varepsilon\to 0^+} \int \frac{\mathrm{d}\omega}{2\pi} \mathrm{e}^{\mathrm{i}\omega\varepsilon} \int \mathrm{d}^2x\, \frac{G_0(\omega;\boldsymbol{x},\boldsymbol{0})G_0(\omega;\boldsymbol{0},\boldsymbol{x})}{\frac{1}{\lambda_R} - \frac{1}{2\pi}\psi\left(\frac{1}{2} - \frac{\omega+\mu}{eB}\right)}. \tag{17}$$

Performing the integral over $\boldsymbol{x}$ with the help of the identity $\int_0^\infty \mathrm{e}^{-z} L_m(z) L_n(z)\,\mathrm{d}z = \delta_{m,n}$ one finds:

$$N_1 = -\frac{\mathrm{i}}{4\pi^2 eB} \lim_{\varepsilon\to 0^+} \int \mathrm{d}\omega\, \mathrm{e}^{\mathrm{i}\omega\varepsilon} \frac{\psi'\left(\frac{1}{2} - \frac{\omega+\mu}{eB}\right)}{\frac{1}{\lambda_R} - \frac{1}{2\pi}\psi\left(\frac{1}{2} - \frac{\omega+\mu}{eB}\right)} \tag{18}$$

where $\psi'(z) = \mathrm{d}\psi(z)/\mathrm{d}z$. The integration over ω can also be performed using residues, but now there are two classes of poles to consider. The poles of the first class have the form $\omega_n^{(1)} = -\mu + (n+\frac{1}{2})eB$ $(n = 0, 1, 2, \ldots)$. They are second-order poles of ψ', but are also simple poles of ψ, and so are simple poles of the integrand. Their contribution to N_1 reads

$$N_1^{(1)} = -\sum_{n=0}^{\infty} \theta(\mu - (n+\tfrac{1}{2})eB). \tag{19}$$

The poles of the second class are given by the roots of the equation

$$\frac{1}{\lambda_R} - \frac{1}{2\pi}\psi\left(\frac{1}{2} - \frac{\omega+\mu}{eB}\right) = 0. \tag{20}$$

Examining the graph of the digamma function [9] one realizes that the roots of equation (20) have the form $\omega_n^{(2)} = -\mu + (k_n + \frac{1}{2})eB$, where $k_0 < 0$ and $n-1 < k_n < n$ $(n = 1, 2, \ldots)$. Their contribution to N_1 has the opposite sign:

$$N_1^{(2)} = \sum_{n=0}^{\infty} \theta(\mu - (k_n+\tfrac{1}{2})eB). \tag{21}$$

Taking into account an area factor A, one finally obtains the following result for $N \equiv \int d^2x\, n(x)$:

$$N = \left(\frac{eB}{2\pi}A - 1\right)\sum_{n=0}^{\infty}\theta(\mu - (n+\tfrac{1}{2})eB) + \sum_{n=0}^{\infty}\theta(\mu - (k_n+\tfrac{1}{2})eB). \tag{22}$$

This result has a very simple physical interpretation: since the potential has zero-range, only the S-waves are affected by it. They are expelled from the Landau levels (which have $eB/2\pi$ states per unit area) and mix among themselves to give new states with energies equal to $(k_n + \frac{1}{2})eB$. The explicit form of their wavefunctions are obtained in the next section.

3.2. Localized states

The wavefunctions of the localized states can also be obtained from the Green function. According to (6) and (13),

$$\begin{aligned}\psi_\ell(\boldsymbol{x})\psi_\ell^*(\boldsymbol{x}') &= \lim_{\omega\to\omega_\ell^{(2)}}(\omega - \omega_\ell^{(2)})G(\omega;\boldsymbol{x},\boldsymbol{x}') \\ &= \frac{2\pi eB}{\psi'(-k_\ell)}G_0(\omega_\ell^{(2)};\boldsymbol{x},\mathbf{0})G_0(\omega_\ell^{(2)};\mathbf{0},\boldsymbol{x}').\end{aligned} \tag{23}$$

It follows from this and equation (9) that

$$\psi_\ell(\boldsymbol{x}) = \sqrt{\frac{eB}{2\pi\psi'(-k_\ell)}}M(\boldsymbol{x},\mathbf{0})\mathrm{e}^{-eB\boldsymbol{x}^2/4}\sum_{n=0}^{\infty}\frac{L_n(eB\boldsymbol{x}^2/2)}{k_\ell - n}. \tag{24}$$

For the lowest-energy bound state ($\ell = 0$), the generating function of Laguerre polynomials, equation (A4), allows us to express the sum in (24) as an integral: since $k_0 < 0$, we may write

$$\begin{aligned}\sum_{n=0}^{\infty}\frac{L_n(\xi)}{k_0 - n} &= -\sum_{n=0}^{\infty}L_n(\xi)\int_0^\infty \mathrm{d}s\, \mathrm{e}^{(k_0-n)s} \\ &= -\int_0^\infty \mathrm{d}s\, \mathrm{e}^{k_0 s}\sum_{n=0}^{\infty}L_n(\xi)\mathrm{e}^{-ns} \\ &= -\int_0^\infty \frac{\mathrm{d}s}{1-\mathrm{e}^{-s}}\exp\left(k_0 s - \frac{\xi}{\mathrm{e}^s - 1}\right).\end{aligned} \tag{25}$$

Changing the variable of integration from s to $z = (\mathrm{e}^s - 1)^{-1}$, we find

$$\sum_{n=0}^{\infty}\frac{L_n(\xi)}{k_0 - n} = -\int_0^\infty \mathrm{e}^{-\xi z}z^{-k_0-1}(1+z)^{k_0}\,\mathrm{d}z = -\Gamma(-k_0)U(-k_0,1,\xi) \tag{26}$$

where $U(a,b,z)$ is the Kummer function which is singular at the origin [9].

Combining (24) and (26) we finally find

$$\psi_0(\boldsymbol{x}) = -\sqrt{\frac{eB}{2\pi\psi'(-k_0)}}\Gamma(-k_0)M(\boldsymbol{x},\mathbf{0})\mathrm{e}^{-eB\boldsymbol{x}^2/4}U(-k_0,1,eB\boldsymbol{x}^2/2). \tag{27}$$

This has precisely the form found by Perez and Coutinho [4] using the method discussed in the introduction.

Although the condition $k_0 < 0$ was essential for obtaining (26), this expression can be analitically continued for $k_0 > 0$ ($k_0 \neq 1, 2, \ldots$), thus generalizing (27) for the other localized states. Note also that, although they diverge at the origin ($U(a,1,z) \approx -[\ln z + \psi(a)]/\Gamma(a)$ when $|z| \to 0$), the wavefunctions $\psi_\ell(\boldsymbol{x})$ are normalizable.

3.3. Hall conductivity

Now, let us consider the Hall conductivity. In the linear response approximation, it is given by [7]

$$\begin{aligned}\sigma_{21}(x) &= \frac{e^2}{2}\lim_{t'\to t+0}(D_{x_2} - D^*_{x'_2})\int \mathrm{d}^3 y\, G(x,y) y_1 G(y,x')\bigg|_{\boldsymbol{x}'=\boldsymbol{x}} \\ &= \frac{e^2}{2}\lim_{\varepsilon\to 0^+}\int \frac{\mathrm{d}\omega}{2\pi}\mathrm{e}^{\mathrm{i}\omega\varepsilon}(D_{x_2} - D^*_{x'_2})\int \mathrm{d}^2 y\, G(\omega;\boldsymbol{x},\boldsymbol{y}) y_1 G(\omega;\boldsymbol{y},\boldsymbol{x}')\bigg|_{\boldsymbol{x}'=\boldsymbol{x}} \qquad (28)\end{aligned}$$

where $\boldsymbol{D} = \boldsymbol{\nabla} - \mathrm{i}e\boldsymbol{A}$ is the gauge-covariant derivative, and $\boldsymbol{D}^*$ is its complex conjugate. After performing the derivatives in x and the integration over $\boldsymbol{y}$ (for the latter, it is useful to use the identities [8] $L_n(z) = L^1_n(z) - L^1_{n-1}(z)$ and $\int_0^\infty z\mathrm{e}^{-z}L^1_m(z)L^1_n(z)\,\mathrm{d}z = (n+1)\delta_{m,n}$), the 'unperturbed' piece of the Hall conductivity, obtained by replacing G with G_0 in (28), reads

$$\sigma^{(00)}_{21} = \frac{\mathrm{i}e^3 B}{8\pi^2}\lim_{\alpha\to 0^+}\lim_{\varepsilon\to 0^+}\int \mathrm{d}\omega\,\mathrm{e}^{\mathrm{i}\omega\varepsilon}\sum_{n=0}^{\infty}[(2n+1)f_n^2 - 2(n+1)f_n f_{n+1}] \qquad (29)$$

where

$$f_n \equiv \frac{\mathrm{e}^{-\alpha n}}{\omega - (n+\frac{1}{2})eB + \mu}. \qquad (30)$$

Performing the remaining integral (along the contour of figure 1), and taking the limits $\varepsilon, \alpha \to 0^+$, one finally obtains

$$\sigma^{(00)}_{21} = -\frac{e^2}{2\pi}\sum_{n=0}^{\infty}\theta(\mu - (n+\tfrac{1}{2})eB). \qquad (31)$$

The 'perturbed' piece of the Hall conductivity, obtained by replacing G with G_1 in equation (28), is easily shown to be zero (the integrand is an odd function of y_1). One can also compute exactly the space average of the 'perturbed–unperturbed' pieces (in which one of the G's in equation (28) is replaced by G_0 and the other by G_1); the calculation is rather tedious (it is sketched in the appendix) but the result is remarkably simple: it is zero.

Since all that remains is the 'unperturbed' piece of σ_{21}, we recover the remarkable result of Prange that the Hall conductivity of a two-dimensional electron gas is not affected by a δ-impurity, even though such an impurity is capable of producing localized states.

As a final remark, note that there is a simple reason why the correction to the Hall conductivity should vanish: as shown by Středa [10], when the chemical potential is in an energy gap the Hall conductivity is given by the following expression:

$$\sigma_{21} = -\frac{ec}{A}\frac{\partial N}{\partial B} \qquad (32)$$

where N is the number of states below the chemical potential μ and A is the area of the system. With N given by equation (22) (remembering that $c = 1$ in our units), it follows from the above expression and from equation (31) that $\sigma_{21} = \sigma^{(00)}_{21}$.

Note added in proof. The Green's function of a particle in a uniform magnetic field in the presence of a short-range impurity was previously obtained by Gesztezy *et al* (Gesztezy F, Holden H and Šeba P 1989 On point interactions in magnetic field systems *Schrödinger Operators, Standard and Non-standard* ed P Exner and P Šeba (Singapore: World Scientific) pp 147–64). We thank Professor Pavel Exner for calling our attention to that paper.

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Appendix

In this appendix we sketch the calculation of the spatial average of $\sigma_{21}^{(01)}(x)$, obtained by replacing the first G in equation (28) with G_0 and the second with G_1. An explicit calculation shows that

$$(D_{x_2} - D^*_{x'_2}) \int \mathrm{d}^2y\, G_0(\omega; \boldsymbol{x}, \boldsymbol{y}) y_1 G_1(\omega; \boldsymbol{y}, \boldsymbol{x}')\bigg|_{\boldsymbol{x}'=\boldsymbol{x}}$$
$$= \frac{eBy_1G_0(\boldsymbol{y}, \boldsymbol{0})}{\frac{1}{\lambda_R} - \frac{1}{2\pi}\psi\left(\frac{1}{2} - \frac{\omega+\mu}{eB}\right)} \left\{ \left[-\mathrm{i}x_1 + \frac{\mathrm{i}y_1}{2} + \frac{y_2}{2} \right] G_0(\boldsymbol{x}, \boldsymbol{y}) G_0(\boldsymbol{0}, \boldsymbol{x}) \right.$$
$$\left. +(x_2 - y_2)\mathcal{G}_0(\boldsymbol{x}, \boldsymbol{y}) G_0(\boldsymbol{0}, \boldsymbol{x}) - x_2 G_0(\boldsymbol{x}, \boldsymbol{y}) \mathcal{G}_0(\boldsymbol{0}, \boldsymbol{x}) \right\} \tag{A1}$$

where $\mathcal{G}_0$ is obtained from G_0 by replacing the Laguerre polynomials $L_n(z)$ by their derivatives with respect to the argument, $L'_n(z)$. It follows from the explicit form of G_0 and $\mathcal{G}_0$ and from the identity $L'_n(z) = -L^1_{n-1}(z)$ that, when calculating $\sigma_{21}^{(01)} \equiv A^{-1}\int \mathrm{d}^2x\, \sigma_{21}^{(01)}(x)$, one has to deal with integrals which, after the change of variables $(\boldsymbol{x}, \boldsymbol{y}) \to \sqrt{eB/2}(\boldsymbol{x}, \boldsymbol{y})$ is performed, have the following form:

$$I^{\alpha\beta\gamma}_{\ell mn}[P(x_i, y_j)] \equiv \int \mathrm{d}^2x\, \mathrm{d}^2y\, \mathrm{e}^{-F(\boldsymbol{x},\boldsymbol{y})} P(x_i, y_j) L^\alpha_\ell(\boldsymbol{x}^2) L^\beta_m((\boldsymbol{x}-\boldsymbol{y})^2) L^\gamma_n(\boldsymbol{y}^2) \tag{A2}$$

where

$$F(\boldsymbol{x}, \boldsymbol{y}) = \mathrm{i}\epsilon_{ij}x_i y_j + \tfrac{1}{2}[\boldsymbol{x}^2 + (\boldsymbol{x}-\boldsymbol{y})^2 + \boldsymbol{y}^2] \tag{A3}$$

and $P(x_i, y_j)$ is a polynomial (of second degree) in x_i, y_j.

With the help of the generating function of Laguerre polynomials [8],

$$(1-t)^{-1-\alpha} \exp\left(\frac{tz}{t-1}\right) = \sum_{n=0}^{\infty} L^\alpha_n(z) t^n \qquad (|t| < 1) \tag{A4}$$

we define another generating function:

$$\mathcal{Z}_{\alpha\beta\gamma}(t, u, v; \boldsymbol{p}, \boldsymbol{q}) \equiv \sum_{\ell,m,n=0}^{\infty} I^{\alpha\beta\gamma}_{\ell mn}[\mathrm{e}^{\boldsymbol{p}\cdot\boldsymbol{x}+\boldsymbol{q}\cdot\boldsymbol{y}}] t^\ell u^m v^n$$
$$= (1-t)^{-1-\alpha}(1-u)^{-1-\beta}(1-v)^{-1-\gamma} \int \mathrm{d}^2x\, \mathrm{d}^2y\, \mathrm{e}^{\boldsymbol{p}\cdot\boldsymbol{x}+\boldsymbol{q}\cdot\boldsymbol{y}-F(\boldsymbol{x},\boldsymbol{y})}$$
$$\times \exp\left\{ \frac{t\boldsymbol{x}^2}{t-1} + \frac{u(\boldsymbol{x}-\boldsymbol{y})^2}{u-1} + \frac{v\boldsymbol{y}^2}{v-1} \right\} \qquad (|t|, |u|, |v| < 1). \tag{A5}$$

Performing the integrals over $\boldsymbol{x}$ and $\boldsymbol{y}$, we obtain

$$\mathcal{Z}_{\alpha\beta\gamma}(t, u, v; \boldsymbol{p}, \boldsymbol{q}) = \frac{4\pi^2\eta\mathrm{e}^{\eta(a\boldsymbol{p}^2+b\boldsymbol{q}^2+c\boldsymbol{p}\cdot\boldsymbol{q}-\mathrm{i}\epsilon_{ij}p_iq_j)}}{(1-t)^{1+\alpha}(1-u)^{1+\beta}(1-v)^{1+\gamma}} \tag{A6}$$

where

$$a = \frac{1-uv}{(1-u)(1-v)} \tag{A7a}$$

$$b = \frac{1 - tu}{(1-t)(1-u)} \tag{A7b}$$

$$c = \frac{1+u}{1-u} \tag{A7c}$$

$$\eta = \frac{(1-t)(1-u)(1-v)}{4(1-tuv)}. \tag{A7d}$$

The integrals in (A2) can then be obtained as the coefficients of the expansion in a power series in t, u and v of

$$P(\partial_{p_i}, \partial_{q_j}) \mathcal{Z}_{\alpha\beta\gamma}(t, u, v; \boldsymbol{p}, \boldsymbol{q})|_{\boldsymbol{p}=\boldsymbol{q}=\boldsymbol{0}}. \tag{A8}$$

The first type of integrals we need to evaluate is $I^{000}_{\ell mn}[-\mathrm{i}x_1 y_1 + (\mathrm{i}y_1^2 + y_1 y_2)/2]$. Following the recipe given above, we obtain

$$\begin{aligned} \sum_{\ell,m,n=0}^{\infty} & I^{000}_{\ell mn}[-\mathrm{i}x_1 y_1 + (\mathrm{i}y_1^2 + y_1 y_2)/2] t^\ell u^m v^n \\ &= \left(-\mathrm{i} \frac{\partial^2}{\partial p_1 \partial q_1} + \frac{\mathrm{i}}{2} \frac{\partial^2}{\partial q_1^2} + \frac{1}{2} \frac{\partial^2}{\partial q_1 \partial q_2} \right) \mathcal{Z}_{000}(t, u, v; \boldsymbol{p}, \boldsymbol{q}) \Big|_{\boldsymbol{p}=\boldsymbol{q}=\boldsymbol{0}} \\ &= \frac{4\pi^2 \eta^2 (-\mathrm{i}c + \mathrm{i}b)}{(1-t)(1-u)(1-v)} = \frac{\mathrm{i}\pi^2 (t-u)(1-v)}{4(1-tuv)^2} \\ &= \frac{\mathrm{i}\pi^2}{4} (t-u)(1-v) \sum_{k=1}^{\infty} (k+1) t^k u^k v^k. \end{aligned} \tag{A9}$$

It follows that

$$I^{000}_{\ell mn}[-\mathrm{i}x_1 y_1 + (\mathrm{i}y_1^2 + y_1 y_2)/2] = \frac{\mathrm{i}\pi^2}{4} [(m+1)\delta_{\ell,m+1}(\delta_{n,m} - \delta_{n,m+1}) - (m \leftrightarrow \ell)] \tag{A10}$$

so that ($f_k \equiv [\omega - (k + \frac{1}{2})eB + \mu]^{-1}$)

$$\sum_{\ell,m,n=0}^{\infty} I^{000}_{\ell mn}[-\mathrm{i}x_1 y_1 + (\mathrm{i}y_1^2 + y_1 y_2)/2] f_\ell f_m f_n = 0. \tag{A11}$$

The other two types of integrals we need can be found in an analogous way:

$$I^{010}_{\ell mn}[x_2 y_1 - y_1 y_2] = \frac{\mathrm{i}\pi^2}{4} (m+1)(\delta_{\ell,m} - \delta_{\ell,m+1})(\delta_{n,m} - \delta_{n,m+1}) \tag{A12}$$

$$I^{100}_{\ell mn}[-x_2 y_1] = -\frac{\mathrm{i}\pi^2}{4} (\ell+1)(\delta_{m,\ell} - \delta_{m,\ell+1})(\delta_{n,\ell} - \delta_{n,\ell+1}). \tag{A13}$$

Therefore,

$$\sum_{\ell,m,n=0}^{\infty} (I^{010}_{\ell,m-1,n}[x_2 y_1 - y_1 y_2] + I^{100}_{\ell-1,m,n}[-x_2 y_1]) f_\ell f_m f_n = 0. \tag{A14}$$

It follows from (A1), (A11), (A14) and the explicit form of G_0 and $\mathcal{G}_0$ that $\sigma^{(01)}_{21} = 0$. An analogous calculation shows that $\sigma^{(10)}_{21} = 0$, too.

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